



PHY · PHYSICS

01. Vectors & Equilibrium

Official PMDC Solved Past Paper Topical Booklet · High-Yield Examination Series

© 50 Solved Questions

2008 to 2025 Archive

Boards: PMDC

Weightage: 27% of MDCAT (54 MCQs)

Curated by Ibrahim for fellow bees. Every question in this chapter has been verified against official UHS, NUMS, SZABMU, DUHS, and KMU answer keys. Propolis cognitive autopsies are provided at the end of this booklet to explain core concepts and break down common distractor traps.

OFFICIAL SOLVED EDITION

Q1.

PMDC

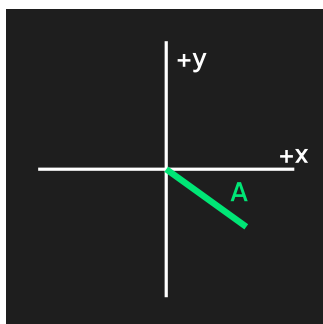
The minimum number of coplanar vectors of unequal magnitude required to produce a zero resultant vector is:

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Q2.

PMDC

If a vector $\vec{A}$ has components $A_x = 1.5$ cm and $A_y = -1.0$ cm, in which quadrant of the Cartesian coordinate system does the vector point?



- (A) Quadrant I
- (B) Quadrant II
- (C) Quadrant III
- (D) Quadrant IV

Q3.

PMDC

Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which of the following forces **cannot** be a resultant of these two forces?

- (A) 40 N
- (B) 30 N
- (C) 20 N
- (D) 70 N

Q4.

PMDC

The sum of the magnitudes of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to the smaller force, then the magnitudes of the individual forces are:

- (A) 6 N and 10 N
- (B) 8 N and 8 N
- (C) 4 N and 12 N
- (D) 2 N and 14 N

Q5.

PMDC

For which angle between two non-zero vectors $\vec{A}$ and $\vec{B}$ is the relation $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ satisfied?

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

Q6.

PMDC

What is the scalar (dot) product of the orthogonal unit vectors $\hat{i}$ and $\hat{j}$?

- (A) 0
(B) 1
(C) -1
(D) $\hat{k}$

Q7.

PMDC

The direction of a vector product (cross product) is determined by which of the following rules?

- (A) Head-to-tail rule
(B) Right-hand rule
(C) Left-hand rule
(D) Fleming's left-hand rule

Q8.

PMDC

What is the cross product of the unit vectors $\hat{i} \times \hat{j}$?

- (A) 0
(B) $-\hat{j}$
(C) $-\hat{k}$
(D) $\hat{k}$

Q9.

PMDC

A rigid body is in a state of complete equilibrium only if which of the following conditions is satisfied?

- (A) $\Sigma \vec{F} = 0$
(B) $\Sigma \vec{\tau} = 0$
(C) $\Sigma \vec{F} = 0$ and $\Sigma \vec{\tau} = 0$
(D) $\Sigma F_x = \Sigma F_y$

Q10.

PMDC

The unit vector $\hat{a}$ in the direction of vector $\vec{a}$ is mathematically defined as:

- (A) $|\vec{a}|$
(B) $\vec{a} \cdot \vec{a}$
(C) $\vec{a} \times \vec{a}$
(D) $\vec{a}/|\vec{a}|$

Q11.

PMDC

Which mathematical condition must be satisfied to guarantee that an object is in rotational equilibrium?

- (A) $\Sigma \vec{\tau} = 0$
(B) $\Sigma \vec{F} = 0$
(C) $\Sigma \vec{F} = 0$ and $\Sigma \vec{\tau} = 0$
(D) $\Sigma \vec{F} > 0$

Q12.

PMDC

Two forces of 5 N and 7 N act on a single point. Which of the following force values **cannot** be their resultant?

- (A) 3 N
(B) 7 N
(C) 11 N
(D) 1 N

Q13.

PMDC

If a force vector is given by $\vec{F} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and the displacement vector is $\vec{D} = \hat{i} - 2\hat{j} - 5\hat{k}$, calculate the work done by the force.

- (A) 28 J
(B) -24 J
(C) -8 J
(D) 16 J

Q14.

PMDC

The sum and the difference of two perpendicular vectors of the same magnitude are always:

- (A) Parallel to each other
(B) Perpendicular to each other
(C) At an acute angle with each other
(D) At an obtuse angle with each other

Q15.

PMDC

A null vector is defined as a vector that has:

- (A) Zero magnitude and a specific direction along the x-axis
(B) Unit magnitude and an arbitrary direction
(C) Zero magnitude and no specific direction
(D) Infinite magnitude and a random direction

Q16.

PMDC

The net external torque acting on a rigid body determines its:

- (A) Linear acceleration
(B) Angular acceleration
(C) Linear momentum
(D) Moment of inertia

Q17.

PMDC

Which of the following physical quantities is a scalar quantity?

- (A) Power
(B) Torque
(C) Momentum
(D) Impulse

Q18.

PMDC

The magnitude of the resultant of two forces is at its minimum value when the angle between them is:

- (A) 0°
- (B) 45°
- (C) 90°
- (D) 180°

Q19.

PMDC

Which of the following pairs contains exactly one vector quantity and one scalar quantity?

- (A) Displacement, Acceleration
- (B) Force, Kinetic Energy
- (C) Power, Speed
- (D) Momentum, Velocity

Q20.

PMDC

Which of the following objects has every point on its surface equidistant from its center of gravity?

- (A) An egg
- (B) A cubic box
- (C) A triangular prism
- (D) A table tennis ball

Q21.

PMDC

A unit vector is primarily used in physics to specify which of the following properties?

- (A) The direction of a vector
- (B) The magnitude of a vector
- (C) The angle of a vector with the origin
- (D) The position of a vector in space

Q22.

PMDC

If the rectangular x-component R_x of a vector is positive and its y-component R_y is negative, in which quadrant does the resultant vector lie?

- (A) First quadrant
- (B) Second quadrant
- (C) Third quadrant
- (D) Fourth quadrant

Q23.

PMDC

Torque is also commonly referred to in physics as the:

- (A) Moment arm
- (B) Moment of force
- (C) Moment of inertia
- (D) Impulse of force

Q24.

PMDC

By standard sign convention, an anticlockwise (counterclockwise) torque is considered:

- (A) Positive
- (B) Negative
- (C) Zero
- (D) Imaginary

Q25.

PMDC

Two equal and opposite forces acting on a body along different lines of action form a:

- (A) Resultant vector
- (B) Moment arm
- (C) Couple
- (D) Linear impulse

Q26.

PMDC

When a rigid body satisfies the first condition of equilibrium ($\Sigma \vec{F} = 0$), it is guaranteed to be in:

- (A) Translational equilibrium
- (B) Rotational equilibrium
- (C) Complete static equilibrium
- (D) Dynamic rotational equilibrium

Q27.

PMDC

A force of magnitude 10 N acts at an angle of 30° relative to the y-axis. What is the magnitude of its rectangular x-component?

- (A) 5.0 N
- (B) 8.66 N
- (C) 10.0 N
- (D) 0.0 N

Q28.

PMDC

The rectangular components of a vector in a 2D plane are equal in magnitude if the angle θ of the vector relative to the x-axis is:

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

Q29.

PMDC

If the position vector $\vec{r}$ of a point of application of force and the force vector $\vec{F}$ are collinear (pointing in the same direction), what is the resulting torque?

- (A) Maximum
- (B) Minimum but non-zero
- (C) Zero
- (D) Negative

Q30.

PMDC

If a rigid body is either at rest or rotating with a constant angular velocity, the net external torque acting on the system must be:

- (A) Maximum
- (B) Negative
- (C) Zero
- (D) Positive

Q31.

PMDC

The area of a parallelogram formed by two vectors $\vec{A}$ and $\vec{B}$ as its adjacent sides is equal to:

- (A) AB
- (B) $AB \cos \theta$
- (C) $AB \sin \theta$
- (D) $AB \tan \theta$

Q32.

PMDC

The magnitude of the dot product of two vectors is $6\sqrt{3}$ and the magnitude of their cross product is 6. What is the angle between the two vectors?

- (A) 0°
- (B) 30°
- (C) 45°
- (D) 60°

Q33.

PMDC

If the cross product of two non-zero vectors $\vec{a}$ and $\vec{b}$ is directed along the z-axis, then both vectors $\vec{a}$ and $\vec{b}$ must lie in the:

- (A) yz-plane
- (B) zx-plane
- (C) xy-plane
- (D) 3D space of any orientation

Q34.

PMDC

Given vectors $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 2\hat{k}$, find a vector $\vec{x}$ that is parallel to $\vec{A}$ but has a magnitude equal to that of $\vec{B}$.

- (A) $\frac{2\sqrt{21}}{7}(2\hat{i} + 3\hat{j} - \hat{k})$
- (B) $\sqrt{\frac{7}{12}}(4\hat{i} + 2\hat{j} - 2\hat{k})$
- (C) $\sqrt{\frac{7}{12}}(2\hat{i} + 3\hat{j} + \hat{k})$
- (D) $\sqrt{\frac{3}{5}}(\hat{i} + 2\hat{j} - \hat{k})$

Q35.

PMDC

A particle undergoes a displacement from position vector $\vec{r}_1 = 3\hat{i} + 2\hat{j} - 6\hat{k}$ to position vector $\vec{r}_2 = 14\hat{i} + 13\hat{j} + 9\hat{k}$ under the action of a constant force $\vec{F} = 8\hat{i} + 2\hat{j} + 6\hat{k}$. Find the work done by the force.

- (A) 50 J
- (B) 125 J
- (C) 155 J
- (D) 200 J

Q36.

PMDC

A vector (such as the velocity of a body undergoing uniform translational motion) that can be displaced parallel to itself without altering its effect is known as a:

- (A) Unit vector
- (B) Position vector
- (C) Free vector
- (D) Null vector

Q37.

PMDC

Determine the unit vector that is parallel to the vector $\vec{B} = 6\hat{i} + 12\hat{j} - 4\hat{k}$.

- (A) $\frac{4}{14}\hat{i} + \frac{12}{14}\hat{j} - \frac{4}{14}\hat{k}$
- (B) $\frac{6}{14}\hat{i} + \frac{12}{14}\hat{j} - \frac{4}{14}\hat{k}$
- (C) $\frac{6}{14}\hat{i} + \frac{10}{14}\hat{j} - \frac{4}{14}\hat{k}$
- (D) $\frac{9}{14}\hat{i} + \frac{12}{14}\hat{j} - \frac{1}{14}\hat{k}$

Q38.

PMDC

Two position vectors $\vec{R}_1$ and $\vec{R}_2$ make angles of 30° and 90° with the positive x-axis, respectively. If $|\vec{R}_1| = 4$ cm and $|\vec{R}_2| = 3$ cm, find the magnitude of their vector product.

- (A) $12\sqrt{3}$ cm²
- (B) $6\sqrt{3}$ cm²
- (C) $6\sqrt{12}$ cm²
- (D) $3\sqrt{6}$ cm²

Q39.

PMDC

Find the work done in moving an object along a straight line from position $(3, 2, -1)$ to position $(2, -1, 4)$ under a force field given by $\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.

- (A) 10 J
- (B) 12 J
- (C) 15 J
- (D) 17 J

Q40.

PMDC

What is the vector projection of $\vec{E} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ onto the direction of vector $\vec{F} = \hat{i} + 2\hat{j} + 2\hat{k}$?

- (A) $1/2$
- (B) $8/3$
- (C) $3/5$
- (D) $5/7$

Q41.

PMDC

Two vectors $\vec{S}$ and $\vec{R}$ have magnitudes of 4 and 6, respectively. If their scalar product is $\vec{S} \cdot \vec{R} = 13.5$, find the angle between the two vectors.

- (A) 22.99°
- (B) 55.77°
- (C) 77.81°
- (D) 99.18°

Q42.

PMDC

The resultant of two forces has a magnitude of 20 N. If one of the forces has a magnitude of $20\sqrt{3}$ N and makes an angle of 30° with the resultant force, what is the magnitude of the other force?

- (A) $10\sqrt{3}$ N
- (B) 20 N
- (C) $20\sqrt{3}$ N
- (D) 10 N

Q43.

PMDC

If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are 5, 4, and 3 units respectively, what is the angle between vector $\vec{A}$ and vector $\vec{C}$?

- (A) $\cos^{-1}(3/5)$
- (B) $\cos^{-1}(4/5)$
- (C) $\sin^{-1}(3/4)$
- (D) 90°

Q44.

PMDC

If the expression $0.8\hat{i} + 0.5\hat{j} + c\hat{k}$ represents a unit vector, find the value of the constant c .

- (A) $\sqrt{0.64}$
- (B) 0.20
- (C) $\sqrt{0.11}$
- (D) 1.00

Q45.

PMDC

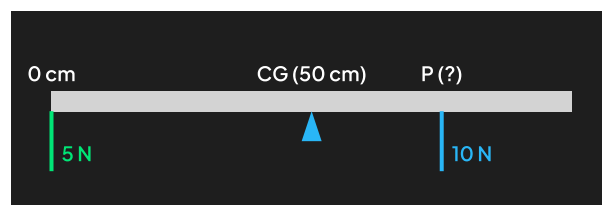
Which of the following descriptions of travel by a car over a one-hour period results in identical average velocities?

- (A) Car 1: Travels 20 km due East; Car 2: Travels 70 km due East
- (B) Car 1: Travels 40 km East, turns around and travels 20 km West; Car 2: Travels 20 km East
- (C) Car 1: Travels 70 km East; Car 2: Travels 40 km East, turns around and travels 20 km West
- (D) Car 1: Travels 30 km West, turns around and travels 30 km East; Car 2: Travels 20 km East

Q46.

PMDC

A uniform 100 cm meter rod is balanced at its center of gravity (the 50 cm mark). A downward force of 5 N is applied at the 0 cm mark. Where must a downward force of 10 N be applied to keep the rod in balance?



- (A) 80 cm mark
- (B) 75 cm mark
- (C) 70 cm mark
- (D) 65 cm mark

Q47.

PMDC

If the rectangular x-component of a vector is $\sqrt{3}$ and its y-component is 1, what is the angle made by the vector with the positive x-axis?

- (A) 60°
- (B) 30°
- (C) 45°
- (D) 90°

Q48.

PMDC

If two vectors $\vec{A}$ and $\vec{B}$ satisfy the equation $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, what is the angle between them?

- (A) 0°
- (B) 60°
- (C) 90°
- (D) 180°

Q49.

PMDC

If $\vec{A} + \vec{B} = 7\hat{i} + 7\hat{k}$ and $\vec{A} - \vec{B} = -\hat{i} + \hat{k}$, what is the scalar magnitude of vector $\vec{A}$?

- (A) 3
- (B) 5
- (C) 7
- (D) 10

Q50.

PMDC

A force couple is formed by two equal and opposite forces that do not share a common line of action.

Which of the following statements about a force couple is correct?

- (A) Its torque depends on the chosen coordinate origin.
- (B) It produces a non-zero net translational force.
- (C) Its net torque is independent of the coordinate origin.
- (D) It can cause translational acceleration of the body.

RAPID 5–COLUMN ANSWER KEY

Official PMDC verified answer keys for 01. Vectors & Equilibrium.

beambeprep.com

Q#	KEY	BOARD	Q#	KEY	BOARD	Q#	KEY	BOARD	Q#	KEY	BOARD	Q#	KEY	BOARD
1	B	PMDC	11	A	PMDC	21	A	PMDC	31	C	PMDC	41	B	PMDC
2	D	PMDC	12	D	PMDC	22	D	PMDC	32	B	PMDC	42	B	PMDC
3	C	PMDC	13	D	PMDC	23	B	PMDC	33	C	PMDC	43	A	PMDC
4	A	PMDC	14	B	PMDC	24	A	PMDC	34	A	PMDC	44	C	PMDC
5	B	PMDC	15	C	PMDC	25	C	PMDC	35	D	PMDC	45	B	PMDC
6	A	PMDC	16	B	PMDC	26	A	PMDC	36	C	PMDC	46	B	PMDC
7	B	PMDC	17	A	PMDC	27	A	PMDC	37	B	PMDC	47	B	PMDC
8	D	PMDC	18	D	PMDC	28	B	PMDC	38	B	PMDC	48	C	PMDC
9	C	PMDC	19	B	PMDC	29	C	PMDC	39	C	PMDC	49	B	PMDC
10	D	PMDC	20	D	PMDC	30	C	PMDC	40	B	PMDC	50	C	PMDC

TOTAL MCQS
50

CORRECT

INCORRECT

BLANK

ACCURACY

Scan the booklet QR code to enter answers in **BeambePrep Grind Mode** for automated error autopsies.

PROPOLIS COGNITIVE ERROR AUTOPSIES

Step-by-step conceptual derivations, formula working, and distractor autopsies for every question.

Question #1 Key: (B)

PMDC

The minimum number of coplanar vectors of unequal magnitude required to produce a zero resultant vector is:

CORE CONCEPT AND DERIVATION

Vector addition and the conditions for a zero resultant vector.

Formula:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} = 0$$

Solution:

- A single non-zero vector can never have a zero resultant because there is no other force to cancel its magnitude.
- Two vectors can only yield a zero resultant if they have equal magnitudes and point in exactly opposite directions. If they are of unequal magnitude, they cannot cancel each other out.
- Three coplanar vectors of unequal magnitude can be arranged such that they form a closed triangle when joined head-to-tail, which makes their vector sum exactly zero.

Why other options are incorrect:

- **A is incorrect** because two unequal vectors will always leave a non-zero net force in the direction of the larger vector.
- **C and D are incorrect** because while 4 or 5 unequal vectors can sum to zero, they do not represent the *minimum* number required.

Question #2 Key: (D)

PMDC

If a vector [Formula] has components [Formula] and [Formula], in which quadrant of the Cartesian coordinate s...

CORE CONCEPT AND DERIVATION

Cartesian Coordinate Quadrants and the signs of rectangular vector components.

Solution:

Let's look at the algebraic signs of the given rectangular components:

- The x-component is positive: $A_x > 0$ (points to the right).
- The y-component is negative: $A_y < 0$ (points downwards).

Aney point with a positive horizontal coordinate and a negative vertical coordinate lies in the fourth quadrant (Quadrant IV).

Why other options are incorrect:

- **Quadrant I** requires both components to be positive ($A_x > 0, A_y > 0$).
- **Quadrant II** requires $A_x < 0, A_y > 0$.
- **Quadrant III** requires both components to be negative ($A_x < 0, A_y < 0$).

Question #3 Key: (C)

PMDC

Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which of the following forces cannot be a ...

CORE CONCEPT AND DERIVATION

The range of possible values for the resultant of two vectors.

Formula:

$$|F_1 - F_2| \leq R \leq F_1 + F_2$$

Solution:

Given $F_1 = 50$ N and $F_2 = 20$ N:

- The minimum possible resultant magnitude occurs when the two forces act in opposite directions ($\theta = 180^\circ$):
Min Resultant = 50 N $-$ 20 N = 30 N
- The maximum possible resultant magnitude occurs when they act in the same direction ($\theta = 0^\circ$):
Max Resultant = 50 N $+$ 20 N = 70 N

The magnitude of the resultant force must fall within the range [30 N, 70 N]. Since 20 N is less than 30 N, it cannot be a possible resultant force.

Why other options are incorrect:

- **30 N** is the minimum limit (opposite direction).
- **40 N** is within the valid range.
- **70 N** is the maximum limit (same direction).

Question #4 Key: (A)

PMDC

The sum of the magnitudes of two forces acting at a point is 16 N. If the resultant force is 8 N and its direc...

CORE CONCEPT AND DERIVATION

Vector addition using right-triangle properties when the resultant is perpendicular to one of the component forces.

Formula:

$$F_1 + F_2 = 16 \text{ N}$$

$$F_1^2 + R^2 = F_2^2$$

Solution:

Let the smaller force be F_1 and the larger force be F_2 . Thus, $F_2 = 16 - F_1$.

Since the resultant $R = 8 \text{ N}$ is perpendicular to F_1 , these vectors form a right-angled triangle where F_2 is the hypotenuse:

$$F_1^2 + 8^2 = (16 - F_1)^2$$

$$F_1^2 + 64 = 256 - 32F_1 + F_1^2$$

$$32F_1 = 256 - 64$$

$$32F_1 = 192 \implies F_1 = 6 \text{ N}$$

Calculating the larger force:

$$F_2 = 16 - 6 = 10 \text{ N}$$

Why other options are incorrect:

- **8 N and 8 N** is incorrect because if both are 8 N, their sum is 16 N but they cannot form a right triangle with a perpendicular resultant of 8 N.
- **4 N and 12 N, and 2 N and 14 N** are incorrect because these pairs do not satisfy the Pythagorean relation with a third side of 8 N. For example, $4^2 + 8^2 = 80 \neq 12^2$.

Question #5 Key: (B)

PMDC

For which angle between two non-zero vectors [Formula] and [Formula] is the relation [Formula] satisfied?

CORE CONCEPT AND DERIVATION

Equating the definitions of the scalar (dot) product and vector (cross) product magnitudes.

Formula:

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

Solution:

Set the two magnitudes equal to each other:

$$AB \cos \theta = AB \sin \theta$$

Since the vectors are non-zero ($A \neq 0, B \neq 0$), we can divide both sides by $AB \cos \theta$:

$$\frac{\sin \theta}{\cos \theta} = 1 \implies \tan \theta = 1$$

$$\theta = \arctan(1) = 45^\circ$$

Why other options are incorrect:

- 30° : $\sin(30^\circ) = 0.5$ while $\cos(30^\circ) = \frac{\sqrt{3}}{2}$ (they are not equal).
- 60° : $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ while $\cos(60^\circ) = 0.5$ (they are not equal).
- 90° : The dot product is zero, and the cross product is at its maximum magnitude.

Question #6 Key: (A)

PMDC

What is the scalar (dot) product of the orthogonal unit vectors [Formula] and [Formula]?

CORE CONCEPT AND DERIVATION

The scalar product of orthogonal vectors.

Formula:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Solution:

The unit vectors $\hat{i}$ and $\hat{j}$ lie along the positive x-axis and y-axis respectively. The angle θ between them is exactly 90° :

$$\hat{i} \cdot \hat{j} = (1)(1) \cos(90^\circ) = 1 \cdot 0 = 0$$

Why other options are incorrect:

- **1 and -1** are incorrect because the dot product of any two distinct, perpendicular unit vectors is always zero, not 1 or -1.
- $\hat{k}$ is incorrect because the scalar product yields a scalar, not a vector (which is the result of the cross product $\hat{i} \times \hat{j}$).

Question #7 Key: (B)

PMDC

The direction of a vector product (cross product) is determined by which of the following rules?

CORE CONCEPT AND DERIVATION

Finding the orientation of the cross-product vector.

Solution:

The direction of $\vec{C} = \vec{A} \times \vec{B}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$. This direction is uniquely determined by the **Right-hand rule**: curl the fingers of your right hand from the first vector $\vec{A}$ to the second vector $\vec{B}$ through the smaller angle; your extended thumb will point in the direction of the product vector $\vec{C}$.

Why other options are incorrect:

- **A is incorrect** because the head-to-tail rule is used for vector *addition*, not multiplication.
- **C and D are incorrect** because left-hand rules do not define standard vector cross-product conventions in mathematics.

Question #8 Key: (D)

PMDC

What is the cross product of the unit vectors [Formula]?

CORE CONCEPT AND DERIVATION

Cyclic permutation of Cartesian unit vectors in cross products.

Solution:

For the orthogonal unit vectors, the cross product of any two yields the third. The standard positive direction follows the cyclic order: $\hat{i} \rightarrow \hat{j} \rightarrow \hat{k} \rightarrow \hat{i}$.
Therefore:

$$\hat{i} \times \hat{j} = \hat{k}$$

Why other options are incorrect:

- **A is incorrect** because $\hat{i}$ and $\hat{j}$ are orthogonal, meaning their cross product is at maximum magnitude (1), not zero.
- **B and C are incorrect** because they represent the wrong directions. Only a positive $\hat{k}$ points in the direction determined by the right-hand rule.

Question #9 Key: (C)

PMDC

A rigid body is in a state of complete equilibrium only if which of the following conditions is satisfied?

CORE CONCEPT AND DERIVATION

The dual requirements for translational and rotational equilibrium.

Formula:

$$\text{Translational Equilibrium: } \Sigma \vec{F} = 0$$

$$\text{Rotational Equilibrium: } \Sigma \vec{\tau} = 0$$

Solution:

For a body to remain in complete equilibrium, it must not undergo translational acceleration nor rotational acceleration. This requires both the net external force and the net external torque about any axis to be zero.

Why other options are incorrect:

- **A is incorrect** because satisfying only the first condition allows the body to undergo angular acceleration.
- **B is incorrect** because satisfying only the second condition allows the body to undergo linear translation.
- **D is incorrect** because the horizontal and vertical forces being equal does not ensure they sum to zero.

Question #10 Key: (D)

PMDC

The unit vector [Formula] in the direction of vector [Formula] is mathematically defined as:

CORE CONCEPT AND DERIVATION

Mathematical definition of a unit vector.

Formula:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

Solution:

A unit vector is a vector that has a magnitude of exactly 1 and points in the same direction as the original vector. It is obtained by dividing the vector $\vec{a}$ by its scalar magnitude $|\vec{a}|$.

Why other options are incorrect:

- **A is incorrect** because $|\vec{a}|$ is a scalar magnitude, not a vector.
- **B is incorrect** because the dot product $\vec{a} \cdot \vec{a}$ yields a scalar equal to $|\vec{a}|^2$.
- **C is incorrect** because the cross product of any vector with itself is always the null vector.

Question #11 Key: (A)

PMDC

Which mathematical condition must be satisfied to guarantee that an object is in rotational equilibrium?

CORE CONCEPT AND DERIVATION

The specific condition for rotational equilibrium.

Formula:

$$\Sigma \vec{\tau} = 0$$

Solution:

Rotational equilibrium means that the angular acceleration of the body is zero. According to Newton's second law for rotation ($\Sigma \vec{\tau} = I\vec{\alpha}$), this state is achieved if and only if the vector sum of all external torques acting on the body is zero.

Why other options are incorrect:

- **B is incorrect** because it defines translational equilibrium, not rotational.
- **C is incorrect** because while it defines *complete* equilibrium, the question specifically asks for the condition of *rotational* equilibrium.
- **D is incorrect** because a non-zero net force has no direct bearing on rotational equilibrium unless it creates a torque.

Question #12 Key: (D)

PMDC

Two forces of 5 N and 7 N act on a single point. Which of the following force values cannot be their resultant...

CORE CONCEPT AND DERIVATION

Vector limits using the triangle inequality theorem.

Formula:

$$|F_1 - F_2| \leq R \leq F_1 + F_2$$

Solution:

With individual magnitudes of 5 N and 7 N:

- The minimum possible resultant magnitude (anti-parallel configuration) is:

$$7 \text{ N} - 5 \text{ N} = 2 \text{ N}$$

- The maximum possible resultant magnitude (parallel configuration) is:

$$7 \text{ N} + 5 \text{ N} = 12 \text{ N}$$

Any value smaller than 2 N or larger than 12 N is physically impossible. Therefore, a resultant of 1 N cannot be formed.

Why other options are incorrect:

- **3 N, 7 N, and 11 N** all fall within the physically permissible interval of [2 N, 12 N].

Question #13 Key: (D)

PMDC

If a force vector is given by [Formula] and the displacement vector is [Formula], calculate the work done by ...

CORE CONCEPT AND DERIVATION

Work defined as the scalar (dot) product of force and displacement.

Formula:

$$W = \vec{F} \cdot \vec{D} = F_x D_x + F_y D_y + F_z D_z$$

Solution:

Substitute the components of the vectors into the formula:

$$W = (2)(1) + (3)(-2) + (-4)(-5)$$

$$W = 2 - 6 + 20$$

$$W = 16 \text{ J}$$

Why other options are incorrect:

- **A, B, and C** result from algebraic sign errors during multiplication (e.g., writing $(-4) \times (-5) = -20$ or incorrectly adding/subtracting the components).

Question #14 Key: (B)

PMDC

The sum and the difference of two perpendicular vectors of the same magnitude are always:

CORE CONCEPT AND DERIVATION

Geometric properties of vector addition and subtraction.

Formula:

Let the perpendicular vectors be $\vec{P}$ and $\vec{Q}$ with $|\vec{P}| = |\vec{Q}| = a$ and $\vec{P} \cdot \vec{Q} = 0$.

$$\vec{S} = \vec{P} + \vec{Q}$$

$$\vec{D} = \vec{P} - \vec{Q}$$

Solution:

Take the scalar product of the sum $\vec{S}$ and the difference $\vec{D}$:

$$\vec{S} \cdot \vec{D} = (\vec{P} + \vec{Q}) \cdot (\vec{P} - \vec{Q})$$

$$\vec{S} \cdot \vec{D} = \vec{P} \cdot \vec{P} - \vec{P} \cdot \vec{Q} + \vec{Q} \cdot \vec{P} - \vec{Q} \cdot \vec{Q}$$

Since $\vec{P} \cdot \vec{Q} = 0$ (perpendicular) and $\vec{P} \cdot \vec{P} = |\vec{P}|^2 = a^2$:

$$\vec{S} \cdot \vec{D} = a^2 - 0 + 0 - a^2 = 0$$

Since their dot product is zero, the sum vector and difference vector are perpendicular to each other.

Why other options are incorrect:

- **A is incorrect** because parallel vectors would require a non-zero dot product equal to the product of their magnitudes.
- **C and D are incorrect** because the dot product is exactly zero, which uniquely corresponds to an angle of 90° .

Question #15 Key: (C)

PMDC

A null vector is defined as a vector that has:

CORE CONCEPT AND DERIVATION

The properties of the null (zero) vector.

Solution:

A null vector (represented as $\vec{0}$) is a vector with a magnitude of exactly zero. Because its length is zero, it does not point in any specific direction, and its direction is mathematically defined as arbitrary or undefined.

Why other options are incorrect:

- **A is incorrect** because a null vector cannot have any specified direction.
- **B and D are incorrect** because they assign non-zero magnitudes (1 and infinity) to the zero vector.

Question #16 Key: (B)

PMDC

The net external torque acting on a rigid body determines its:

CORE CONCEPT AND DERIVATION

Newton's second law for rotational motion.

Formula:

$$\vec{\tau} = I\vec{\alpha}$$

Solution:

Just as net force determines the linear acceleration of a mass ($\vec{F} = m\vec{a}$), the net external torque $\vec{\tau}$ acting on a body determines its angular acceleration $\vec{\alpha}$.

Why other options are incorrect:

- **A and C are incorrect** because linear acceleration and linear momentum are governed by translational forces, not torques.
- **D is incorrect** because the moment of inertia is an intrinsic geometric property of the mass distribution, independent of the applied torque.

Question #17 Key: (A)

PMDC

Which of the following physical quantities is a scalar quantity?

CORE CONCEPT AND DERIVATION

Classification of scalar and vector quantities.

Solution:

Power is the rate at which work is done. It has only magnitude and does not possess a direction, making it a scalar quantity.

Why other options are incorrect:

- **Torque** is a vector pointing perpendicular to the plane of rotation.
- **Momentum** is a vector pointing in the direction of the velocity vector.
- **Impulse** is a vector representing the change in momentum (pointing in the direction of the net force).

Question #18 Key: (D)

PMDC

The magnitude of the resultant of two forces is at its minimum value when the angle between them is:

CORE CONCEPT AND DERIVATION

Law of cosines for vector addition and its extrema.

Formula:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

Solution:

To minimize R , we need to minimize $\cos \theta$. The minimum value of the cosine function is -1 , which occurs when:

$$\theta = 180^\circ$$

In this case:

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2} = \sqrt{(F_1 - F_2)^2} = |F_1 - F_2|$$

This matches the scenario where the forces point in opposite directions.

Why other options are incorrect:

- 0° yields the maximum possible resultant ($F_1 + F_2$).
- 90° yields $\sqrt{F_1^2 + F_2^2}$, which is greater than the minimum difference.
- 45° yields an intermediate value.

Question #19 Key: (B)

PMDC

Which of the following pairs contains exactly one vector quantity and one scalar quantity?

CORE CONCEPT AND DERIVATION

Identifying vectors and scalars within physical quantity pairings.

Solution:

Let's analyze each pair:

- Force is a vector quantity (magnitude and direction), whereas Kinetic Energy is a scalar quantity (only magnitude). This satisfies the requirement of exactly one vector and one scalar.

Why other options are incorrect:

- **A:** Displacement and acceleration are both vectors.
- **C:** Power and speed are both scalars.
- **D:** Momentum and velocity are both vectors.

Question #20 Key: (D)

PMDC

Which of the following objects has every point on its surface equidistant from its center of gravity?

CORE CONCEPT AND DERIVATION

Uniform mass distribution, geometric centers, and centers of gravity of regular shapes.

Solution:

A table tennis ball is highly spherical and has a uniform mass distribution. For a uniform sphere, the center of gravity coincides with its geometric center. Because every point on the surface of a sphere is equidistant from its center, the table tennis ball perfectly satisfies this property.

Why other options are incorrect:

- **An egg** is ellipsoidal, so surface points at the poles are further from the center of gravity than points at the equator.
- **A cubic box and a triangular prism** have vertices and flat faces, meaning corners are much further from the center of gravity than the center of the faces.

Question #21 Key: (A)

PMDC

A unit vector is primarily used in physics to specify which of the following properties?

CORE CONCEPT AND DERIVATION

The purpose of unit vectors.

Solution:

Unit vectors have a magnitude of exactly 1. Multiplying any scalar value by a unit vector assigns a direction to that value without changing its magnitude. Thus, they are used to describe direction.

Why other options are incorrect:

- **B is incorrect** because the magnitude of a unit vector is always fixed at 1, so it cannot describe the variable magnitude of other vectors.
- **C and D are incorrect** because a unit vector alone does not specify position in space or reference angles.

Question #22 Key: (D)

PMDC

If the rectangular x -component [Formula] of a vector is positive and its y -component [Formula] is negative, in...

CORE CONCEPT AND DERIVATION

Quadrant determination using component signs.

Solution:

The signs of the components determine the direction:

- $R_x > 0$ (points along the positive x -axis, to the right)
- $R_y < 0$ (points along the negative y -axis, downwards)

The right-down direction corresponds to the fourth quadrant.

Why other options are incorrect:

- **First quadrant:** Requires both components to be positive.
- **Second quadrant:** Requires negative x -component and positive y -component.
- **Third quadrant:** Requires both components to be negative.

Question #23 Key: (B)

PMDC

Torque is also commonly referred to in physics as the:

CORE CONCEPT AND DERIVATION

Scientific nomenclature for rotational force effects.

Solution:

Torque measures the tendency of a force to rotate an object about an axis. In classical mechanics, this rotational effect is also called the **moment of force**.

Why other options are incorrect:

- **Moment arm** is the perpendicular distance from the axis of rotation to the line of action of the force.
- **Moment of inertia** is a measure of an object's resistance to rotational acceleration.
- **Impulse** is the product of force and the time interval over which it acts.

Question #24 Key: (A)

PMDC

By standard sign convention, an anticlockwise (counterclockwise) torque is considered:

CORE CONCEPT AND DERIVATION

Standard rotational sign conventions.

Solution:

By standard mathematical convention (the right-hand rule applied to the coordinate plane), rotations and torques in the anticlockwise direction are defined as **positive**, whereas clockwise rotations are defined as **negative**.

Why other options are incorrect:

- **Negative** is the sign convention reserved for clockwise torques.
- **Zero and Imaginary** are incorrect because active torques have non-zero real values.

Question #25 Key: (C)

PMDC

Two equal and opposite forces acting on a body along different lines of action form a:

CORE CONCEPT AND DERIVATION

Definition and properties of a force couple.

Solution:

A **couple** is defined as two parallel forces that are equal in magnitude, opposite in direction, and do not share a common line of action. A couple produces a pure torque with zero net translational force.

Why other options are incorrect:

- **Resultant vector:** The net vector sum of these two forces is actually zero, so they do not produce a net translational force.
- **Moment arm:** This is a distance measurement, not a system of forces.
- **Linear impulse:** This is the change in linear momentum, which is zero since the net force of a couple is zero.

Question #26 Key: (A)

PMDC

When a rigid body satisfies the first condition of equilibrium ([Formula]), it is guaranteed to be in:

CORE CONCEPT AND DERIVATION

The first condition of equilibrium and its physical consequence.

Solution:

The first condition of equilibrium states that the vector sum of all forces acting on a body must be zero ($\Sigma \vec{F} = 0$). According to Newton's second law ($\Sigma \vec{F} = m\vec{a}$), this ensures that the translational acceleration of the body's center of mass is zero. Therefore, the body is in **translational equilibrium**.

Why other options are incorrect:

- **Rotational equilibrium** requires the sum of all torques to be zero (the second condition).
- **Complete static equilibrium** requires both translational and rotational equilibrium to be satisfied, and the body must be at rest.

Question #27 Key: (A)

PMDC

A force of magnitude 10 N acts at an angle of 30° relative to the y-axis. What is the magnitude of its rectangular component along the x-axis?

CORE CONCEPT AND DERIVATION

Resolving a vector using an angle defined relative to the vertical (y-axis).

Formula:

$$F_x = F \sin \theta_y$$

Solution:

When the angle $\theta = 30^\circ$ is measured relative to the y-axis, the horizontal (x) component is associated with the sine of that angle:

$$F_x = F \sin(30^\circ)$$

$$F_x = 10 \text{ N} \times 0.5 = 5.0 \text{ N}$$

Why other options are incorrect:

- **8.66 N** is the vertical (y) component, calculated using $10 \cos(30^\circ) = 10 \times 0.866 = 8.66 \text{ N}$.
- **10.0 N** is the total magnitude of the force.

Question #28 Key: (B)

PMDC

The rectangular components of a vector in a 2D plane are equal in magnitude if the angle [Formula] of the vector is...

CORE CONCEPT AND DERIVATION

Equality of rectangular components.

Formula:

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

Solution:

For the magnitudes to be equal ($|A_x| = |A_y|$):

$$A \cos \theta = A \sin \theta \implies \tan \theta = 1$$

For angles in the first quadrant, this yields:

$$\theta = 45^\circ$$

Why other options are incorrect:

- 30° : The horizontal component is larger ($\cos 30^\circ \approx 0.866 > \sin 30^\circ = 0.5$).
- 60° : The vertical component is larger ($\sin 60^\circ \approx 0.866 > \cos 60^\circ = 0.5$).
- 90° : The horizontal component is zero, and the vertical component is equal to the vector's full magnitude.

Question #29 Key: (C)

PMDC

If the position vector [Formula] of a point of application of force and the force vector [Formula] are collinear...

CORE CONCEPT AND DERIVATION

Calculating torque when the force is parallel to the displacement vector.

Formula:

$$\vec{\tau} = \vec{r} \times \vec{F} \implies \tau = rF \sin \theta$$

Solution:

If the vectors point in the same direction, the angle θ between them is 0° .

$$\tau = rF \sin(0^\circ) = rF(0) = 0$$

Therefore, the torque is exactly zero.

Why other options are incorrect:

- **A is incorrect** because the maximum torque occurs when the force is perpendicular to the position vector ($\theta = 90^\circ$).
- **B and D are incorrect** because the sine of 0° is zero, making the torque zero.

Question #30 Key: (C)

PMDC

If a rigid body is either at rest or rotating with a constant angular velocity, the net external torque acting on it is...

CORE CONCEPT AND DERIVATION

Rotational equilibrium and its relation to torque.

Formula:

$$\vec{\tau}_{\text{net}} = I\vec{\alpha}$$

Solution:

If a body is at rest or rotating with a constant angular velocity, its angular acceleration $\vec{\alpha}$ is zero:

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = 0$$

Substituting this into Newton's rotational second law yields:

$$\vec{\tau}_{\text{net}} = I(0) = 0$$

Why other options are incorrect:

- **A, B, and D** are incorrect because any non-zero net torque (whether positive, negative, or maximum) would produce an angular acceleration, causing the angular velocity to change over time.

Question #31 Key: (C)

PMDC

The area of a parallelogram formed by two vectors [Formula] and [Formula] as its adjacent sides is equal to:

CORE CONCEPT AND DERIVATION

Geometric interpretation of the magnitude of the vector cross product.

Formula:

$$\text{Area} = |\vec{A} \times \vec{B}| = AB \sin \theta$$

Solution:

The base of the parallelogram is A . The height is the perpendicular component of vector $\vec{B}$, which is given by $B \sin \theta$.

$$\text{Area} = \text{base} \times \text{height} = A(B \sin \theta) = AB \sin \theta$$

This is exactly equal to the magnitude of the cross product of the two vectors.

Why other options are incorrect:

- AB is the area of a rectangle with sides of length A and B .
- $AB \cos \theta$ is the scalar dot product of the two vectors.
- $AB \tan \theta$ does not correspond to any standard geometric area for these vectors.

Question #32 Key: (B)

PMDC

The magnitude of the dot product of two vectors is [Formula] and the magnitude of their cross product is 6. Wh...

CORE CONCEPT AND DERIVATION

Finding the angle between vectors using the ratio of cross and dot products.

Formula:

$$\vec{A} \cdot \vec{B} = AB \cos \theta = 6\sqrt{3}$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta = 6$$

Solution:

Divide the magnitude of the cross product by the dot product:

$$\frac{AB \sin \theta}{AB \cos \theta} = \frac{6}{6\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Why other options are incorrect:

- 0° : The cross product would be zero.
- 45° : The dot product and cross product would be equal in magnitude.
- 60° : This would swap the two values, making the dot product 6 and the cross product $6\sqrt{3}$.

Question #33 Key: (C)

PMDC

If the cross product of two non-zero vectors [Formula] and [Formula] is directed along the z-axis, then both v...

CORE CONCEPT AND DERIVATION

The perpendicular nature of the vector cross product.

Formula:

$$\vec{c} = \vec{a} \times \vec{b} \implies \vec{c} \perp \vec{a} \text{ and } \vec{c} \perp \vec{b}$$

Solution:

The vector cross product produces a vector that is always perpendicular to the plane containing the original vectors. If the resulting vector points along the z-axis, then both $\vec{a}$ and $\vec{b}$ must be perpendicular to the z-axis. This means they must lie within the flat plane perpendicular to the z-axis, which is the xy-plane.

Why other options are incorrect:

- **yz-plane** contains vectors whose cross products must point along the x-axis.
- **zx-plane** contains vectors whose cross products must point along the y-axis.

Question #34 Key: (A)

PMDC

Given vectors [Formula] and [Formula], find a vector [Formula] that is parallel to [Formula] but has a magnit...

CORE CONCEPT AND DERIVATION

Scaling a unit vector to construct a vector with a specific direction and magnitude.

Formula:

$$\vec{x} = |\vec{B}| \hat{A} = |\vec{B}| \frac{\vec{A}}{|\vec{A}|}$$

Solution:

First, calculate the scalar magnitude of both vectors:

$$|\vec{B}| = \sqrt{4^2 + 2^2 + (-2)^2} = \sqrt{16 + 4 + 4} = \sqrt{24}$$

$$|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

Now substitute these values into our equation for $\vec{x}$:

$$\vec{x} = \sqrt{24} \frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}} = \sqrt{\frac{24}{14}} (2\hat{i} + 3\hat{j} - \hat{k})$$

Simplify the radical fraction:

$$\sqrt{\frac{24}{14}} = \sqrt{\frac{12}{7}} = \sqrt{\frac{12 \times 7}{7^2}} = \frac{\sqrt{84}}{7} = \frac{2\sqrt{21}}{7}$$

Therefore, the constructed vector is:

$$\vec{x} = \frac{2\sqrt{21}}{7} (2\hat{i} + 3\hat{j} - \hat{k})$$

Why other options are incorrect:

- **B, C, and D** use incorrect magnitude ratios or incorrect vector components that do not point in the direction of $\vec{A}$.

Question #35 Key: (D)

PMDC

A particle undergoes a displacement from position vector [Formula] to position vector [Formula] under the acti...

CORE CONCEPT AND DERIVATION

Work done calculated using the displacement vector and the force vector.

Formula:

$$\vec{d} = \vec{r}_2 - \vec{r}_1$$

$$W = \vec{F} \cdot \vec{d}$$

Solution:

Step 1: Calculate the displacement vector $\vec{d}$:

$$\vec{d} = (14 - 3)\hat{i} + (13 - 2)\hat{j} + (9 - (-6))\hat{k}$$

$$\vec{d} = 11\hat{i} + 11\hat{j} + 15\hat{k}$$

Step 2: Calculate the dot product with the force vector:

$$W = \vec{F} \cdot \vec{d} = (8)(11) + (2)(11) + (6)(15)$$

$$W = 88 + 22 + 90$$

$$W = 200 \text{ J}$$

Why other options are incorrect:

- **A, B, and C** are incorrect values that result from mathematical errors when calculating the displacement coordinates (e.g. evaluating $9 - 6 = 3$ instead of $9 - (-6) = 15$).

Question #36 Key: (C)

PMDC

A vector (such as the velocity of a body undergoing uniform translational motion) that can be displaced parall...

CORE CONCEPT AND DERIVATION

Categorization of vectors based on localization.

Solution:

A **free vector** is a vector whose point of application is not restricted to a specific point in space. It can be translated parallel to itself to any location without changing its physical meaning or mathematical representation (such as velocity or displacement).

Why other options are incorrect:

- **Unit vector** is defined strictly by having a magnitude of one.
- **Position vector** is tied to a specific point relative to a chosen coordinate origin.
- **Null vector** has a magnitude of zero and no direction.

Question #37 Key: (B)

PMDC

Determine the unit vector that is parallel to the vector [Formula].

CORE CONCEPT AND DERIVATION

Constructing a unit vector from a Cartesian component vector.

Formula:

$$\hat{B} = \frac{\vec{B}}{|\vec{B}|}$$

Solution:

Step 1: Calculate the scalar magnitude of the vector:

$$|\vec{B}| = \sqrt{6^2 + 12^2 + (-4)^2}$$

$$|\vec{B}| = \sqrt{36 + 144 + 16} = \sqrt{196} = 14$$

Step 2: Divide each component of the vector by this magnitude:

$$\hat{B} = \frac{6}{14}\hat{i} + \frac{12}{14}\hat{j} - \frac{4}{14}\hat{k}$$

Why other options are incorrect:

- **A, C, and D** are incorrect because their components do not match those of the original vector $\vec{B}$.

Question #38 Key: (B)

PMDC

Two position vectors [Formula] and [Formula] make angles of 30° and 90° with the positive x-axis, respectively...

CORE CONCEPT AND DERIVATION

Calculating the magnitude of a cross product using the angle between two vectors.

Formula:

$$|\vec{R}_1 \times \vec{R}_2| = R_1 R_2 \sin \theta$$

Solution:

First, determine the angle θ between the two vectors:

$$\theta = \theta_2 - \theta_1 = 90^\circ - 30^\circ = 60^\circ$$

Now substitute the magnitudes and the angle into the formula:

$$|\vec{R}_1 \times \vec{R}_2| = (4 \text{ cm})(3 \text{ cm}) \sin(60^\circ)$$

$$|\vec{R}_1 \times \vec{R}_2| = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ cm}^2$$

Why other options are incorrect:

- **A is incorrect** because it omits the factor of $1/2$ from the sine value (using 1 instead of $\sin 60^\circ$).
- **C and D** are mathematically incorrect results from applying the wrong trigonometric functions or angles.

Question #39 Key: (C)

PMDC

Find the work done in moving an object along a straight line from position [Formula] to position [Formula] and...

CORE CONCEPT AND DERIVATION

Calculating work done between two spatial coordinates.

Formula:

$$\vec{d} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$W = \vec{F} \cdot \vec{d}$$

Solution:

Step 1: Calculate the displacement vector $\vec{d}$:

$$\vec{d} = (2 - 3)\hat{i} + (-1 - 2)\hat{j} + (4 - (-1))\hat{k}$$

$$\vec{d} = -\hat{i} - 3\hat{j} + 5\hat{k}$$

Step 2: Calculate the dot product:

$$W = (4)(-1) + (-3)(-3) + (2)(5)$$

$$W = -4 + 9 + 10$$

$$W = 15 \text{ J}$$

Why other options are incorrect:

- **A, B, and D** result from arithmetic mistakes such as writing $4 - (-1) = 3$ or failing to include the negative sign for the first component.

Question #40 Key: (B)

PMDC

What is the vector projection of [Formula] onto the direction of vector [Formula]?

CORE CONCEPT AND DERIVATION

Finding the scalar projection of one vector onto another.

Formula:

$$\text{Projection} = \frac{\vec{E} \cdot \vec{F}}{|\vec{F}|}$$

Solution:

Step 1: Calculate the dot product $\vec{E} \cdot \vec{F}$:

$$\vec{E} \cdot \vec{F} = (2)(1) + (-3)(2) + (6)(2) = 2 - 6 + 12 = 8$$

Step 2: Calculate the magnitude of the target vector $\vec{F}$:

$$|\vec{F}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$$

Step 3: Evaluate the projection:

$$\text{Projection} = \frac{8}{3}$$

Why other options are incorrect:

- **A, C, and D** are incorrect fractions that result from arithmetic errors in the dot product or using the wrong vector's magnitude in the denominator.

Question #41 Key: (B)

PMDC

Two vectors [Formula] and [Formula] have magnitudes of 4 and 6, respectively. If their scalar product is [Form...]

CORE CONCEPT AND DERIVATION

Calculating the angle between vectors using the definition of the dot product.

Formula:

$$\vec{S} \cdot \vec{R} = SR \cos \theta \implies \cos \theta = \frac{\vec{S} \cdot \vec{R}}{SR}$$

Solution:

Substitute the given values into the equation:

$$\cos \theta = \frac{13.5}{4 \times 6} = \frac{13.5}{24} = 0.5625$$

$$\theta = \arccos(0.5625) \approx 55.77^\circ$$

Why other options are incorrect:

- **A, C, and D** are incorrect angles that result from arithmetic calculation errors, such as dividing by the sum of the magnitudes (10) instead of their product (24).

Question #42 Key: (B)

PMDC

The resultant of two forces has a magnitude of 20 N. If one of the forces has a magnitude of [Formula] and mak...

CORE CONCEPT AND DERIVATION

Solving for a component vector magnitude using the law of cosines.

Formula:

$$B^2 = A^2 + R^2 - 2AR \cos \phi$$

Solution:

Let the resultant force be $R = 20$ N and the known force be $A = 20\sqrt{3}$ N. The angle between them is $\phi = 30^\circ$.

Using the law of cosines on the vector triangle:

$$B^2 = (20\sqrt{3})^2 + (20)^2 - 2(20\sqrt{3})(20) \cos(30^\circ)$$

$$B^2 = (400 \times 3) + 400 - 800\sqrt{3} \left(\frac{\sqrt{3}}{2}\right)$$

$$B^2 = 1200 + 400 - 1200$$

$$B^2 = 400 \implies B = 20 \text{ N}$$

Why other options are incorrect:

- **A, C, and D** are mathematically incorrect and do not satisfy the law of cosines for this vector geometry.

Question #43 Key: (A)

PMDC

If [Formula] and the magnitudes of vectors [Formula], [Formula], and [Formula] are 5, 4, and 3 units respect...

CORE CONCEPT AND DERIVATION

Using right-angled triangle properties to find the angle between vector components.

Solution:

The magnitudes of the vectors (5, 4, and 3) satisfy the Pythagorean theorem:

$$5^2 = 4^2 + 3^2 \implies 25 = 16 + 9$$

This means vectors $\vec{B}$ and $\vec{C}$ are perpendicular to each other, and $\vec{A}$ forms the hypotenuse of a right-angled triangle.

The angle θ between $\vec{A}$ (the hypotenuse) and $\vec{C}$ (the adjacent side) is given by:

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{C}{A} = \frac{3}{5}$$

$$\theta = \cos^{-1}(3/5)$$

Why other options are incorrect:

- **B is incorrect** because it defines the angle between the hypotenuse $\vec{A}$ and the vector $\vec{B}$.
- **C and D** are mathematically incorrect and do not match the geometric configuration of this right-angled triangle.

Question #44 Key: (C)

PMDC

If the expression [Formula] represents a unit vector, find the value of the constant [Formula].

CORE CONCEPT AND DERIVATION

The magnitude of a unit vector is exactly equal to one.

Formula:

$$\sqrt{x^2 + y^2 + z^2} = 1$$

Solution:

Set the magnitude of the given vector to 1:

$$\sqrt{(0.8)^2 + (0.5)^2 + c^2} = 1$$

Square both sides of the equation:

$$0.64 + 0.25 + c^2 = 1$$

$$0.89 + c^2 = 1$$

$$c^2 = 1 - 0.89 = 0.11 \implies c = \sqrt{0.11}$$

Why other options are incorrect:

- **A is incorrect** because it equals 0.8, which is the value of the x-component.
- **B and D** are mathematically incorrect and do not satisfy the unit magnitude condition.

Question #45 **Key: (B)**

PMDC

Which of the following descriptions of travel by a car over a one-hour period results in identical average vel...

CORE CONCEPT AND DERIVATION

Average velocity depends on the net displacement vector rather than the total distance traveled.

Formula:

$$\vec{v}_{\text{avg}} = \frac{\vec{d}_{\text{net}}}{t}$$

Solution:

Let's define East as the positive x-direction and West as the negative x-direction. Each trip occurs over a duration of $t = 1$ hour.

Let's calculate the net displacement for the journeys described in **Option B**:

- **Car 1:** Travels +40 km, then -20 km:

$$\vec{d}_{\text{net1}} = 40 \text{ km} - 20 \text{ km} = +20 \text{ km (East)}$$

$$\vec{v}_{\text{avg1}} = \frac{20 \text{ km}}{1 \text{ hr}} = 20 \text{ km/hr East}$$

- **Car 2:** Travels +20 km:

$$\vec{d}_{\text{net2}} = +20 \text{ km (East)}$$

$$\vec{v}_{\text{avg2}} = \frac{20 \text{ km}}{1 \text{ hr}} = 20 \text{ km/hr East}$$

Both cars have the exact same net displacement vector over the same time interval, so they have identical average velocities.

Why other options are incorrect:

- **A, C, and D** contain pairs of journeys with unequal net displacements (for example, in D, Car 1 has a net displacement of 0 km, whereas Car 2 has a net displacement of 20 km).

Question #46 **Key: (B)**

PMDC

A uniform 100 cm meter rod is balanced at its center of gravity (the 50 cm mark). A downward force of 5 N is a...

CORE CONCEPT AND DERIVATION

Using rotational equilibrium to balance torque about a pivot.

Formula:

$$\Sigma \tau = 0 \implies \tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

Solution:

The pivot is at the 50 cm mark. Let's calculate the moment arms:

- The 5 N force is at the 0 cm mark. Its moment arm is:

$$d_1 = 50 \text{ cm} - 0 \text{ cm} = 50 \text{ cm}$$

- This force produces an anticlockwise torque:

$$\tau_{\text{anticlockwise}} = 5 \text{ N} \times 50 \text{ cm} = 250 \text{ N} \cdot \text{cm}$$

- The 10 N force must produce an equal clockwise torque of $250 \text{ N} \cdot \text{cm}$ on the opposite side of the pivot:

$$\tau_{\text{clockwise}} = 10 \text{ N} \times d_2 = 250 \text{ N} \cdot \text{cm}$$

$$d_2 = 25 \text{ cm}$$

Since the clockwise torque must be on the right side of the pivot, the position of this force is:

$$\text{Position} = 50 \text{ cm} + 25 \text{ cm} = 75 \text{ cm mark}$$

Why other options are incorrect:

- **A, C, and D** do not satisfy the condition of rotational equilibrium. If placed at these marks, the clockwise and anticlockwise torques would be unbalanced, causing the rod to tilt.

Question #47 Key: (B)

PMDC

If the rectangular x-component of a vector is [Formula] and its y-component is 1, what is the angle made by th...

CORE CONCEPT AND DERIVATION

Calculating vector direction from its rectangular components.

Formula:

$$\tan \theta = \frac{A_y}{A_x}$$

Solution:

Substitute the component values into the formula:

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Why other options are incorrect:

- 60° : This angle would swap the components, making the x-component 1 and the y-component $\sqrt{3}$.
- 45° : Requires equal components (e.g. 1 and 1).
- 90° : Requires the x-component to be zero.

Question #48 Key: (C)

PMDC

If two vectors [Formula] and [Formula] satisfy the equation [Formula], what is the angle between them?

CORE CONCEPT AND DERIVATION

Relating vector sum and difference magnitudes to the angle between the vectors.

Formula:

$$|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB \cos \theta$$

$$|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB \cos \theta$$

Solution:

Set the squared magnitudes equal to each other:

$$A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

Subtract $A^2 + B^2$ from both sides:

$$2AB \cos \theta = -2AB \cos \theta$$

$$4AB \cos \theta = 0$$

Since the vectors are non-zero ($A \neq 0, B \neq 0$):

$$\cos \theta = 0 \implies \theta = 90^\circ$$

Why other options are incorrect:

- 0° : The sum is at a maximum magnitude ($A + B$), and the difference is at a minimum magnitude ($A - B$).
- 180° : The sum is at a minimum magnitude, and the difference is at a maximum magnitude.
- 60° : The cosine of 60° is 0.5, which does not satisfy the equality.

Question #49 Key: (B)

PMDC

If $\vec{A} = 3\hat{i} + 4\hat{k}$ and $\vec{B} = 7\hat{i} + 7\hat{k}$, what is the scalar magnitude of vector $\vec{A} - \vec{B}$?

CORE CONCEPT AND DERIVATION

Solving systems of vector equations.

Solution:

Step 1: Add the two vector equations together to eliminate $\vec{B}$:

$$(\vec{A} + \vec{B}) + (\vec{A} - \vec{B}) = (7\hat{i} + 7\hat{k}) + (-7\hat{i} + 7\hat{k})$$

$$2\vec{A} = 6\hat{i} + 8\hat{k}$$

$$\vec{A} = 3\hat{i} + 4\hat{k}$$

Step 2: Calculate the scalar magnitude of vector $\vec{A}$:

$$|\vec{A}| = \sqrt{3^2 + 0^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Why other options are incorrect:

- **3, 7, and 10** are incorrect magnitudes that result from mathematical errors (such as using $3 + 4 = 7$ instead of the square root of the sum of squares, or failing to divide by 2).

Question #50 Key: (C)

PMDC

A force couple is formed by two equal and opposite forces that do not share a common line of action. Which of the following is true?

CORE CONCEPT AND DERIVATION

Properties of a force couple.

Solution:

A couple consists of two forces of equal magnitude pointing in opposite directions ($\vec{F}_1 = -\vec{F}_2$).

The net force is exactly zero:

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = 0$$

Because the net force is zero, the torque produced by a couple is constant and has the same value about any point in space, making it completely independent of the choice of coordinate origin.

Why other options are incorrect:

- **A is incorrect** because the origin does not affect the torque calculation for a couple.
- **B and D are incorrect** because the net force is zero, which means there is no translational acceleration.